

# Interactive Learning Module for Control Interaction Understanding

J.L. Guzmán, K.J. Åström, T. Hägglund, S. Dormido, M. Berenguel, and Y. Pigué

**Abstract**—This paper describes an interactive tool focused on teaching and learning basic concepts on multivariable control systems. Most industrial processes are represented by multivariable systems and thus the teaching and learning of basic concepts on multivariable control is a key factor in control engineering education. In this sense, the interactive tool presented in this paper is aimed to introduce basic ideas on multivariable control systems mainly focused on interaction and pairing concepts.

## I. INTRODUCTION

Most industrial plants are composed of many output variables and many input variables. These systems are known as multivariable systems or multiple inputs and multiple outputs (MIMO) systems. In many situations, each output can be associated with one input in such a way that the full process can be controlled by many single loops in parallel. This is a common solution used in industry where PID controllers are used to control the different single loops. There are, however, situations when there may be considerable interaction between different control loops. Because of the interactions it may be difficult to control such systems loop by loop [2].

These ideas are what we desire to transmit in this paper. Multivariable control theory is a key element to be included in automatic control courses, being a fundamental part in the education of control engineers. There are many different books dedicated to this fact [1], [14], [15] and in the literature it is also possible to find some tools related to show ideas about multivariable control techniques [8], [4]. However, most of this material cover advanced aspects of multivariable control or is focused on specific control techniques. Thus, the idea of this work is to combine basic ideas on multivariable control extracted from [2] with an interactive tool as support to control education. The objective is to introduce the concept of interaction, to show how this interaction can be measured, and how this measurement can be used to decide if a system can be controlled using single loops with PID controllers and which is the best pairing between outputs and control signals. For this purpose, the most simple case for a multivariable

system, which is composed by two inputs and two outputs (TITO), has been selected for a better understanding.

The tool presented in this paper is part of a project called *Interactive Learning Modules*. During the writing of the book [2] the idea emerged that it would be useful to try to develop interactive learning modules (ILMs) for modelling and control. The idea is to develop interactive learning tools which could be used for introductory control courses at universities and other schools, and for engineers in industry. The modules should be self-contained, they should be suitable both for self-study, for courses, and for demonstrations in lectures, and they should not require any additional software. In this sense, we have been working in this project for three years developing different interactive tools focused on PID Control [7], Dead-time control [9], and Modelling and Identification [6]. Therefore, the interactive tool described here is attempted to continue this contribution about interactive tools for the control education area.

The interactive tool presented in this paper has been implemented in Sysquake [12]. The main reasons for choosing Sysquake was its power to develop interactive graphical tools and the possibility to generate executable files that can run independently and distributed without any licences.

The idea to change properties and immediately being able to see the effects of the changes is very powerful both for learning and for designing. The dynamics of the changes provides additional information that is not available in static plots. There are many interesting issues that have to be dealt with when developing interactive tools for the automatic control field which are related to the particular graphics representations used. It is straight forward to see the effects of parameters on the graphics but not so obvious how the graphical objects should be manipulated. There are natural ways to modify pole-zero plots for example by adding poles and zeros and by dragging them. Bode plots can be manipulated by dragging the intersections of the asymptotes. However, it is less obvious how a Nyquist plot should be changed [10].

One consideration that it must be taken into account with the use of the interactive tools is that the power of these interactive features sometimes provokes the user's temptation to play with the different dynamic objects forgetting the real meaning of such graphical elements. Teachers must make clear to the students that interactive tools are abstractions of the theoretical concepts, and the full learning must be complemented as a mixture of both [10]. For that reason, interactive tools are recommended as support to consolidated theoretical books looking for this combination between theory and practice. The ILM project is a clear example of that

This work has been financed by CICYT-FEDER under Grants DPI2007-66718-C04-04 and DPI2007-61068, and by the IV PRICIT (Plan Regional de Ciencia y Tecnología de la Comunidad de Madrid), under Grant S-0505/DPI/0391.

J.L. Guzmán and M. Berenguel are with Automatics and Control Engineering Area, Department of Computer Science, University of Almería, 04120 Almería, Spain {joguzman,beren}@ual.es

K.J. Åström and T. Hägglund are with Department of Automatic Control, Lund University, Box 118 SE-22100 Lund, Sweden {kja,tore}@control.lth.se

S. Dormido is with Dep. Informática y Automática, ETSI Informática, UNED, 28040 Madrid, Spain sdormido@dia.uned.es

Y. Pigué is with Calerga Sàrl, 35, av. de la Chablière, 1004 Lausanne, Switzerland yves.pigué@calerga.com

where interactive tools are directly related with the book [2].

Notice that this new interactive learning module has been developed recently, and feedback from the students is not yet available. The module will be used in different control courses at Lund University, University of Almería, UNED, and EPFL. One consideration that must be kept in mind is that the tool's main feature, interactivity, cannot be easily illustrated in a written text. Nevertheless, some of the advantages of the applications are shown in the paper. The reader is cordially invited to visit the web site (at [www.calerga.com](http://www.calerga.com) and [aer.ual.es/ilm](http://aer.ual.es/ilm)) to experience the interactive features of the module.

The paper is organized as follows. In Section 2, basic ideas about the effects of the interaction, measurement for interaction, and pairing are given. The interactive tool is presented in Section 3, and a few examples are given in Section 4. The paper ends with conclusions and references.

## II. THEORETICAL BASIS

As described above, the ideas and theory presented in this paper will be focused on TITO systems for a better understanding. A general system with two inputs and two outputs can be represented by the following equations [2]:

$$\begin{aligned} Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\ Y_2(s) &= p_{21}(s)U_1(s) + p_{22}U_2(s), \end{aligned} \quad (1)$$

where  $Y_i$  and  $U_i$  are the Laplace transforms of process output  $y_i$  and process input  $u_i$ , respectively, and  $p_{ij}(s)$  is the transfer function from the  $j$ :th input to the  $i$ :th output. The transfer functions  $p_{11}$ ,  $p_{12}(s)$ ,  $p_{21}(s)$ , and  $p_{22}$  can be combined into the matrix

$$P(s) = \begin{pmatrix} p_{11}(s) & p_{12}(s) \\ p_{21}(s) & p_{22}(s) \end{pmatrix}, \quad (2)$$

which is called the transfer function or the matrix transfer function of the system.

In the following sections, elemental concepts and ideas about multivariable control are described, namely, interaction, relative gain array, and pairing.

### A. Interaction

When multivariable control systems are introduced in a control engineering course, the first concept to be studied is *interaction*. Multivariable systems are characterized by the presence of multiple variables which influence each other. This influence between variables is what we understand as interaction from a multivariable control point of view. That is, how the control of one variable affects the rest of the control loops existing in the system.

From an education point of view, the best way to understand the effects of interaction is by means of an example. So, an example extracted from [2] is used for this purpose. The system is described by

$$\begin{aligned} Y_1(s) &= \frac{1}{(s+1)^2}U_1(s) + \frac{2}{(s+1)^2}U_2(s) \\ Y_2(s) &= \frac{1}{(s+1)^2}U_1(s) + \frac{1}{(s+1)^2}U_2(s). \end{aligned}$$

Two controllers are used, the controller  $C_1$  controls the output  $y_1$  by the input  $u_1$  and  $C_2$  controls the output  $y_2$  by the input  $u_2$ . One effect of interaction is that the tuning of one loop can influence the other loop. This effect can be observed with the following simple analysis.

The feedback in the second loop is  $U_2(s) = -k_2Y_2(s)$ . Introducing this in the second equation gives

$$U_2(s) = -\frac{k_2}{s^2 + 2s + k_2 + 1}U_1(s),$$

and insertion of this expression for  $U_2(s)$  in the first equation gives

$$Y_1(s) = g_{11}^{cl}(s)U_1(s) = \frac{s^2 + 2s + 1 - k_2}{(s+1)^2(s^2 + 2s + 1 + k_2)}U_1(s).$$

This equation shows clearly that the gain  $k_2$  in the second loop has a significant effect on the dynamics relating  $u_1$  and  $y_1$ . The static gain is

$$g_{11}^{cl}(0) = \frac{1 - k_2}{1 + k_2}.$$

Notice that the gain decreases as  $k_2$  increases and that the gain becomes negative for  $k_2 > 1$ .

The example shows that the gain of the second loop has a significant influence on the behavior of the first loop. The response of the first loop is good when the second loop is disconnected,  $k_2 = 0$ , but the system becomes more sluggish when the gain of the second loop is increased. The system is unstable for  $k_2 \geq 0.8$ .

This example has been included in the interactive tool as described in the following sections. The next section briefly describes a simple method to measure interaction, which will be useful to determine if a multivariable control system is difficult or not to control due to influence effects between variables.

### B. Measuring interaction

The most well-know method to measure interaction is the Relative Gain Array (RGA) introduced by Bristol [2]. This method consists in studying how the static process gain of one loop is influenced by the gains in the other loops. For a system with  $N$  loops, it can be supposed that  $N - 1$  loops are in perfect control, that is, they have reached the desired value in steady-state, and thus, its effect on the remaining loops can be studied. This can be better understood from chapter 11 in [2], where this study is provided by a TITO system.

Consider the system described by (1). Assume that the second loop is in perfect control, and thus that the output of the second loop is zero. Then

$$\begin{aligned} Y_1(s) &= p_{11}(s)U_1(s) + p_{12}U_2(s) \\ 0 &= p_{21}(s)U_1(s) + p_{22}U_2(s). \end{aligned}$$

Eliminating  $U_2(s)$  from the first equation gives

$$Y_1(s) = \frac{p_{11}(s)p_{22}(s) - p_{12}(s)p_{21}(s)}{p_{22}(s)}U_1(s).$$

The ratio of the static gains of loop 1 when the second loop is open and when the second loop is closed is thus

$$\lambda = \frac{p_{11}(0)p_{22}(0)}{p_{11}(0)p_{22}(0) - p_{12}(0)p_{21}(0)}. \quad (3)$$

where parameter  $\lambda$  is called *Bristol's interaction index* for TITO systems. Although we focus on TITO systems in this paper, the calculation of this index can be generalized for systems with many inputs and many outputs, comparing the interaction between all the variables and resulting in the Bristol's Relative Gain Array, know as RGA.

For a TITO system as (1), the RGA becomes [2]

$$R = \begin{pmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{pmatrix}, \quad (4)$$

where  $\lambda$  is the interaction index (3). There is no interaction if  $\lambda = 1$ . This means that the second loop has no impact on the first loop and vice versa. If  $\lambda$  is between 0 and 1 the closed loop has higher gain than the open loop. The effect is most severe for  $\lambda = 0.5$ . If  $\lambda$  is larger than 1 the closed loop has lower gain than the open loop. When  $\lambda$  is negative the gain of the first loop changes sign when the second loop is closed. The effect of the interactions is thus severe.

The capabilities of the interactive tool can be used to clearly show this idea playing with the static gain of the system.

### C. Pairing

In the introduction section, it was mentioned that many multivariable systems in industry are controlled loop by loop. For a proper performance of this idea, a deep analysis of the system must be done looking for the less possible interaction between variables. That is, to control a system loop by loop we must first decide how the controllers should be connected in order to reduce the interaction as much as possible. This choice is known as pairing, which consist in deciding which input should be used to control which output.

As described in the previos section, the RGA allows to determine if the interaction between control loops is severe or not, and thus, this measurement can be used as a guide for pairing. Notice that from the analysis performed above, if  $\lambda = 1$  there is no interaction considering that outputs  $y_1$  and  $y_2$  are controlled by  $u_1$  and  $u_2$ , respectively. If  $\lambda = 0$  there is also no interaction, but the loops should be interchanged. The loops should be interchanged when  $\lambda < 0.5$ . If  $0 < \lambda < 1$  the gain of the first loop increases when the second loop is closed, and if  $\lambda > 1$  the closed-loop gain is less than the open-loop gain. Bristol recommended that pairing should be made so that the corresponding relative gains are positive and as close to one as possible. Pairing of signals with negative relative gains should be avoided. If the gains are outside the interval  $0.67 < \lambda < 1.5$ , other techniques as decoupling can improve the control significantly [2].

Notice that, the structure of the PID controllers used in the interactive tool is the following [2]:

$$U(s) = K \left( bY_{sp} - Y + \frac{1}{sT_i} E - \frac{sT_d}{1 + sT_d/N} Y \right), \quad (5)$$

where  $U$ ,  $Y_{sp}$ ,  $Y$  and  $E$  are the Laplace transforms of control signal  $u$ , setpoint  $y_{sp}$ , process output  $y$ , and control error  $e = y_{sp} - y$ , respectively. Controllers of the types P, I, PD, PI and PID can be chosen and their parameters can be changed via menus or sliders.

## III. INTERACTIVE TOOL

One of the most important features for developing interactive tools is the *abstraction*. In the context of this work, abstraction refers to associate theoretical concepts with graphical elements. That is, visualizing the concepts with graphical and dynamic pictures, knowing this as visualization. In order to design technical systems or simply to understand the physical laws that describe their behavior, scientists and engineers often use computers to calculate and graphically represent different magnitudes. In control engineering, these quantities include among others: time and frequency responses, poles and zeros on the complex plane, Bode, Nyquist and Nichols diagrams, phase plane, etc. Frequently these magnitudes are closely related and constitute different visions of a single reality. The understanding of these relationships is one of the keys to achieve a good learning of the basic concepts and it enables students to accurately design control systems [5]. The tool presented in this paper has been developed following this idea, as shown below.

### A. Description

This section briefly describes the main features of the developed tool. The tool is focused on understanding the basic concepts on multivariable control described in the previous section. As commented before, this tool has been implemented in Sysquake, a Matlab-like language with fast execution and excellent facilities for interactive graphics [12]. Windows, Mac, and Linux operating systems are supported. Remember that the tool's main feature -interactivity- cannot be easily shown in a written text. Therefore, users are cordially invited to experience the interactive features of the tool.

This interactive learning module has been developed to make it possible to quickly obtain a good intuition and a good working knowledge of basic multivariable control concepts. As the previous modules developed [7][9][6], the module is composed by several parts. Two icons can be accessed, namely, *Instructions* and *Theory*. *Instructions* gives access to a document that contains suggestions for exercises, while *Theory* provides access to relevant theory by means of the Internet. Different menus are available to load examples and choose between different configurations of the tool. The graphics can be manipulated directly by dragging points, lines, and curves or by using text-edits and sliders. Notice also that for all the graphics available in the tool, the vertical

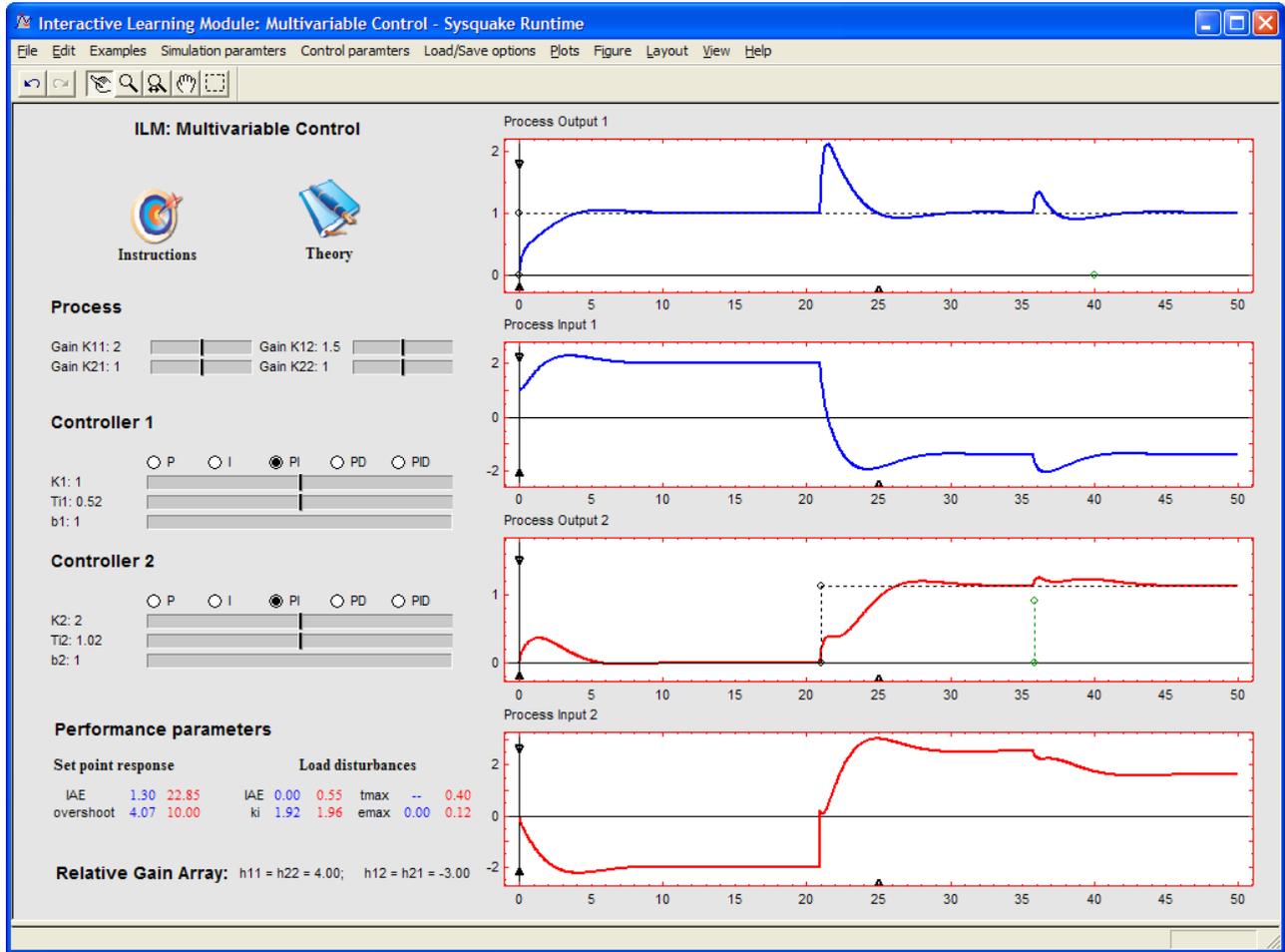


Fig. 1. Interactive tool user interface.

and horizontal scales can be modified using three black triangles available on the graphics ( $\blacktriangle$ ,  $\blacktriangledown$ ).

When developing a tool of this kind, one of the most important things that the developer has to have in mind is the organization of the main windows and menus of the tool to facilitate the user the understanding of the control technique [5], [10]. The main window of the tool is divided into several sections represented in Figure 1 (basic screen of the developed interactive tool) which are described as follows

- *Time-domain graphics.* There exist four graphical elements on the right side of the screen, which represent the system outputs (Process Output 1 and Process Output 2) and the controller outputs (Process Input 1 and Process Input 2). The graphics show the simulation results of the control algorithm selected for a step change in set-points and load disturbance (in the inputs).

Over these graphics the user can interactively modify the amplitudes of the set-points and the disturbances dragging on the black and green dots, and also to change the scales dragging on the small black triangles.

- *Process and control parameters.* The different parameters available in the tool are shown on the left-hand side of the screen (see Figure 1). These elements allow to modify process parameters, the controller type, and controller parameters. So, it is possible to modify the process parameter for each transfer function of the matrix transfer function in the form (2). For each controller,  $C_1$  and  $C_2$ , the controller parameters described in the structure (5) can be modified interactively.
- *Performance and interaction parameters.* Some parameters to characterize performance are available in the tool. Performance information with respect to set-point

and disturbance responses is provided just below the process and control parameters for each control loop. Furthermore, in this part of the tool, the RGA for the selected system is permanently shown.

- *Menu options.* Several menu options can be chosen from menu bars. There are four menu entries with specific parameters of the tool: **Examples**, **Simulation parameters**, **Control parameters**, and **Load/Save**. The rest of menu entries are provided by *Sysquake*, the programming environment using to develop the tool. The **Examples** menu allows to choose between typical plants to be studied in the interactive tool. Some of these examples are described in the following section. Furthermore, it is possible to load examples from the book *Advanced PID Control* [2], which the user can explore by modifying parameters. The **Simulation parameters** menu allows to modify the sampling time and also disable the use of load disturbances or graphics autoscale. From the **Control parameters** menu, it is possible to configure different options and parameters for the proposed control schemes. For instance, the option **Switching pairing** is used to switch between the controllers. This option is very useful for pairing when the RGA indicates that the loops should be interchanged. Finally, the **Load/Save options** menu is used to stored and recalled the results using the **Save design** and **Load design** dialogs.

Finally, notice that the simulations in the interactive tool consist of solving linear differential equations with constant coefficients and the complete system is sampled at constant sampling rate and the sampled equations are iterated. Hence, the choice of the sampling time from the **Simulation** menu must be in accordance with the closed-loop system dynamics.

#### IV. ILLUSTRATIVE EXAMPLES

In this section, two examples will illustrate some of the features of the new tool.

##### A. Effects of interaction

As described above, interaction is the first concept that should be introduced in a multivariable control course. This phenomenon can easily be shown using the interactive features of the tool playing around with the different parameters of the system. In this way, the example described in section II-A is available in the tool at the **Examples** menu. By default, a PI controller is given for  $C_1$  with  $k_1 = 1$  and  $k_{i,1} = 1$  and a P controller for  $C_2$  with  $k_2 = 0$  obtaining the results shown in Figure 2 for a step change in the set-point for  $y_1$ . As it can be observed, the output for  $y_2$  is given by the interaction with  $y_1$ .

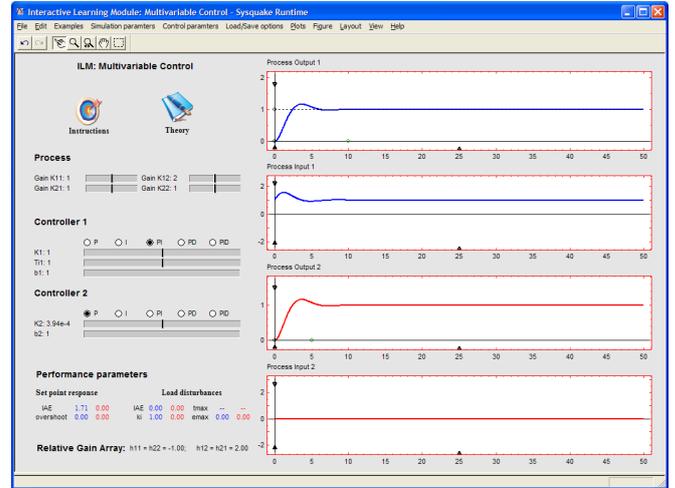


Fig. 2. Example for understanding interaction. System used in section II-A where  $C_1$  is a PI controller with  $k_1 = 1$  and  $k_{i,1} = 1$  and  $C_2$  a proportional controller with  $k_2 = 0$

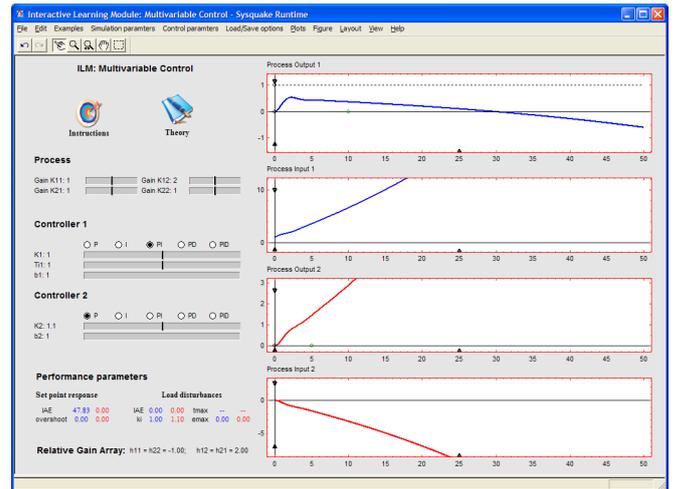


Fig. 3. Example for understanding interaction. System used in section II-A where  $C_1$  is a PI controller with  $k_1 = 1$  and  $k_{i,1} = 1$  and  $C_2$  a proportional controller with  $k_2 = 1.1$

In section II-A, it was deduced that  $k_2$  has a negative effect on the static gain in the control loop 1,  $g_{11}^{cl}(0)$ . The systems becomes unstable for  $k_2 \geq 0.8$ . The tool can be used for this study and to obtain this unstable behavior. For instance, Figure 3 shows the unstable results when  $k_2 = 1.1$ .

##### B. RGA and pairing

Following the ideas discussed in section II-C, the RGA can be used as a guide for pairing. If the RGA is calculated

for the example described in section II-A, it is obtained that

$$R = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix},$$

which means that  $\lambda = -1$  and thus the pairing rule says that  $y_1$  should be paired with  $u_2$ .

Therefore, a bad pairing was performed in the previous example, obtaining a very severe interaction between variables which even made the system unstable. If the **Switching pairing** option from the **Control parameters** menu is used to interchange the loops, that is, controlling  $y_1$  and  $y_2$  with  $u_2$  and  $u_1$ , respectively, it can be seen that the interaction is not as severe as before. Now, the static gain in the control loop 1 decreases with increasing  $k_2$ , but it is never negative for  $k_2 > 0$ . This example can be interactively studied using the tool choosing the pairing example from the **Examples** menu. Figure 4 shows this example, where the same control parameters as in Figure 3 are used, but now the system doesn't become unstable for  $k_2 = 1.1$ .

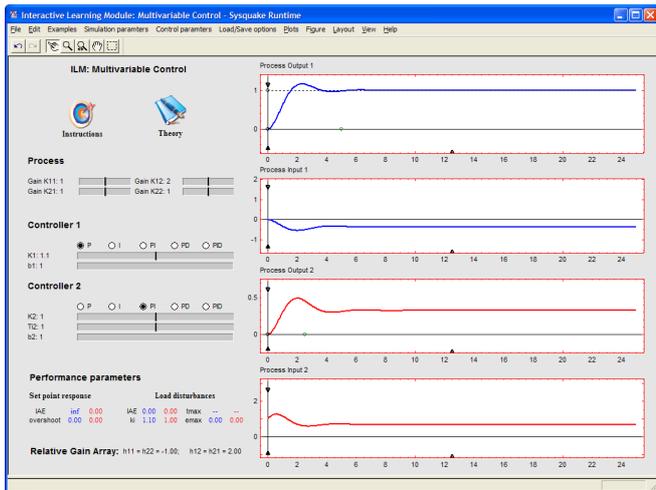


Fig. 4. Example for understanding pairing. System used in Figure 3 switching the control loops.

### C. Other examples

Some other didactical well-known examples are also available in the **Examples** menu of the tool [3]:

- Rosenbrock's system [13].
- Wood-Berry's binary distillation column [16].
- Quadruple tank-process [11].

## V. CONCLUSIONS

A new interactive learning module for basic concepts on multivariable control has been described. The module belongs to the collection of interactive learning modules that are an attempt to make figures in the book [2] interactive. The purpose is to enhance learning by exploiting the advantages of immediately seeing the effects of changes that can never be shown in static pictures. The module is implemented in Sysquake and is available on the web at [www.calerga.com](http://www.calerga.com). Additional modules for design and feedforward are under development.

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