INTERACTIVE TOOL FOR ANALYSIS OF TIME-DELAY SYSTEMS WITH DEAD-TIME COMPENSATORS

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ABSTRACT

This paper presents an interactive tool which allows to study the problem of systems with large delays. The developed tool provides several examples in order to compare PI control and Smith Predictor (SP) approaches for processes with delay. Robustness problems due to presence of model uncertainties can also be studied, as well as other control structures to face the different problems that the original Smith Predictor has with integrator and unstable processes.

1. INTRODUCTION

The term interactivity is being more common each day in the field of teaching in Automatic Control. In general terms, interactivity could only be understood as the response to some action performed by the user. However, looking at the results obtained in the field of Automatic Control this term seems more ambitious (Dormido et al. (2002), Dormido (2004), Guzmán et al. (2005)). At the moment, interactivity is associated with a set of graphical representations, elements and parameters interconnecting each others, in such way that if some element is modified the rest are updated immediately and only spending few seconds. This powerful element has provided very important new possibilities in researching and teaching, being possible to study problems at different complexity levels and using a new interactive philosophy relatively difficult some years ago. Nowadays a new generation of software packages has appeared. Some of them are based on objects that admit a direct graphic manipulation and are automatically updated, so that the relationship among them is continuously maintained. Ictools and CCSdemo (Johansson et al. (1998), Wittenmark et al. (1998)), developed in the Department of Automatic Control at Lund Institute of Technology or SysQuake in the Institut d'Automátique of the Federal Polytechnic School of Lausanne (Piguet, 2004) are good examples of this new educational philosophy applied to the field of automatic control.

Therefore, with the aim of continuing working in this field an interactive tool to study the problems of systems with large delays is presented.

Many processes present delay in their input or output variables. In many cases we can represent their behavior by means of a transfer function described as:

$$G_p(s) = G(s)e^{-\tau s} \qquad (1)$$

Conventional controllers, such as PID controllers (Åström and Hägglund (2005)), could be used when the dead-time is small but they show poor performance when the process exhibits long dead-times since a significant amount of detuning is required to maintain closed-loop stability (Hägglund (1996), Åström and Hägglund (2005)). In these cases, it is convenient to use a dead-time compensating method.

The Smith Predictor (Smith (1959)) and its many extensions, generically named as Process-Model Control schemes (Watanabe and Ito (1981)), can be considered as the first control methods for single-input-singleoutput linear processes showing a delay in their input or output (Gu and Niculescu (2003)). The main advantage of the Smith predictor method is that time-delay is eliminated from the characteristic equation of the closed loop system. Thus, the design and analysis problem for the process with delay can be converted to the one without delay (Åström and Hägglund (2005)). On the other hand, it is well-known that the Smith Predictor presents several problems in presence of integrator or unstable processes, existing available some modifications in order to face them (Åström et al. (1994), Matausek and Micié (1999), Lu et al. (2005)).

So, taking the previous description as motivation, an interactive tool to study the problems of systems with large delays has been developed in this work. The developed tool allows to work with different control algorithms, compare them, and study the stability and performance in an interactive way.

The paper is organized as follows. The next sections briefly describes the Smith Predictor scheme and its variants to treat with integrate and unstable systems. Section 3 presents the developed interactive tool showing its main features. Some examples are shown in section 4 using the presented tool. Finally, conclusions are outlined in section 5.

2. CONTROL SCHEMES TO CONTROL TIME DELAY SYSTEMS

2.1 *Smith Predictor for stable systems*

As commented above, the Smith Predictor (Smith (1959)) is the best know and most widely used algorithm for dead-time compensation (DT-compensators). The typical scheme for this approach is shown in Figure 1.

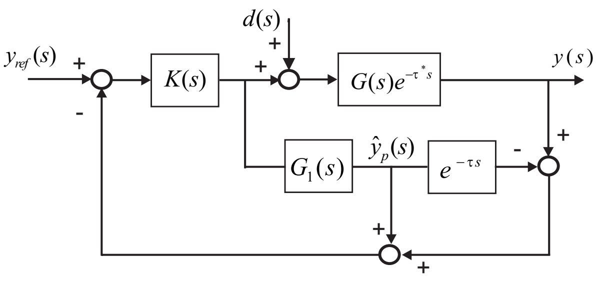

Fig. 1. Smith Predictor scheme.

From this scheme, the following expressions can be obtained:

$$y(s) = \frac{K(s)G(s)e^{-\tau s}}{1+K(s)\left(\hat{G}(s)-\hat{G}(s)e^{-\hat{\tau}s}+G(s)e^{-\tau s}\right)}y_{ref}(s) + \frac{(1+K(s)\hat{G}(s)-K(s)\hat{G}(s)e^{-\hat{\tau}s})G(s)e^{-\tau s}}{1+K(s)\left(\hat{G}(s)-\hat{G}(s)e^{-\hat{\tau}s}+G(s)e^{-\tau s}\right)}d(s) \tag{2}$$

where $G(s)e^{-\tau s}$ represents the real process, $K(s)$ any conventional controller, and $\hat{G}(s)$ and $e^{-\hat{\tau}s}$ the process model free of delay and the delay model respectively.

If the process and delay models are considered to loyally represent the real system, $\hat{G}(s)e^{-\hat{\tau}s} = G(s)e^{-\tau s}$, then the closed-loop response to set-point and disturbance inputs is given by:

$$y(s) = \frac{K(s)G(s)e^{-\tau s}}{1+K(s)G(s)}y_{ref}(s) + \left(1-\frac{K(s)G(s)e^{-\tau s}}{1+K(s)G(s)}\right)G(s)e^{-\tau s}d(s) \tag{3}$$

Remark 1. Note in (3) that, even in the ideal case if the system $G(s)$ has any unstable pole, the closed-loop system will also be unstable.

From some easy algebraic manipulation it is possible rewriting the transfer function (3) as:

$$G_{yd}(s) = \frac{y(s)}{d(s)} = \frac{G(s)e^{-\tau s}}{1+K(s)G(s)} + \frac{K(s)G(s)e^{-\tau s}}{1+K(s)G(s)}(G(s)-G(s)e^{-\tau s})d(s) \tag{4}$$

Remark 2. Note that, the closed-loop system has zero steady-state to step load disturbances only if $\lim_{s\longrightarrow 0}(G(s)-G(s)e^{-\tau s}) = 0$. However, when a system with an integrating mode $G(s) = \frac{G_s(s)}{s}$ is considered, the steady-state error is not zero:

$$\lim_{s\longrightarrow 0}(G(s)-G(s)e^{-\tau s}) = \lim_{s\longrightarrow 0}G_s(s)\tau \tag{5}$$

2.2 *DT-compensators for integrating systems*

Simple models are very important in the process industry, and most processes are modelled by low order transfer functions plus a dead-time, or by an integrator model plus a dead-time (IPDT): $G_p(s) = \frac{K_p}{s}e^{-\tau s}$.

As was discussed in Remark 2, the original Smith Predictor scheme presents some problems for systems with integrating mode in presence of disturbances. In order to solve this problem, some SP-based compensators have been proposed (Åström et al. (1994), Watanabe and Ito (1981), Matausek and Micié (1996)). For this tool the variant developed by (Matausek and Micié (1999)) have been implemented. This approach is based on the idea that the dead-time part not only represents the delay but also the high order dynamic of the real process. Figure 2 shows the scheme of this approach where: $F(s) = \frac{K_0(T_d s+1)}{(T_f s+1)}$, $T_f = \frac{T_d}{10}$.

Remark 3. Note that if $T_d = 0$, then $F(s) = K_0$, and this structure is equivalent to those presented by (Matausek and Micié (1996)). On the other hand, if $F(s) = 0$, the typical Smith Predictor is obtained.

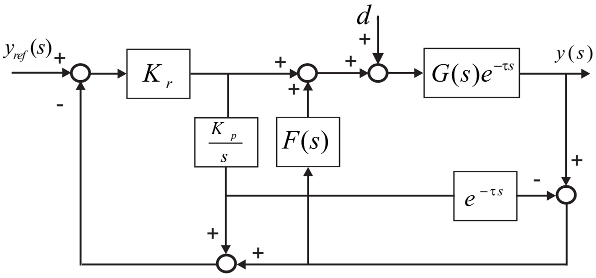

Fig. 2. Matausek DTC structure.

The closed-loop transfer function from the set-point to the output is given by:

$$G_{ry} = \frac{K_r \Delta(s) G(s) e^{-\tau s}}{1 + K_r \frac{K_p}{s} - K_r \frac{K_p}{s} e^{\hat{\tau}} + K_r \Delta(s) G(s) e^{-\tau s}} \quad (6)$$

where $\Delta(s) = \frac{(1+K_0 \hat{G}(s) e^{\hat{\tau} s})}{(1+K_0 G(s) e^{\tau s})}$

When modelling errors are neglected, the following expressions are obtained:

$$y(s) = \frac{K_p K_r e^{-\tau s}}{(s + K_p K_r)} y_{ref}(s) + \frac{K_p \left[s + K_p K_r (1 - e^{-\tau s}) \right] e^{-\tau s}}{(s + K_p K_r)(s + \frac{K_o K_p (T_d s + 1)}{(T_f s + 1)} e^{-\tau s})} d(s) \quad (7)$$

Remark 4. It can be seen that the steady-state error is zero from $\lim_{s \longrightarrow 0} (s + K_p K_r (1 - e^{-\tau s})) = 0$

The tuning of K_r parameter is performed following the formula $K_r = \frac{1}{K_p T_r}$, where T_r is the desired closed-loop time constant (Matausek and Micié (1996)). The tuning of the parameters K_0 and T_d depend on the stability in (7). In (Matausek and Micié (1999)), the tuning rules for these parameters are:

$$T_d = \alpha \tau$$
$$K_0 = \frac{\frac{\pi}{2} - \Phi_{pm}}{K_p \tau \sqrt{(1-\alpha)^2 + (\frac{\pi}{2} - \Phi_{pm})^2 \alpha^2)}} \quad (8)$$

where the values of the parameters $\alpha = 0.5$ and the phase margin $\Phi_{pm} = 64^0$ are determined by simulations (Matausek and Micié (1999)).

2.3 *DT-compensators for unstable systems*

Many different approaches to control time-delay unstable systems have been reported in the literature. Recently, in (Lu et al. (2005)) has been presented a scheme (Figure 3) which has already demonstrated its superiority over previous approaches.

In the nominal case, the closed-loop response to set-point and disturbance inputs is given by:

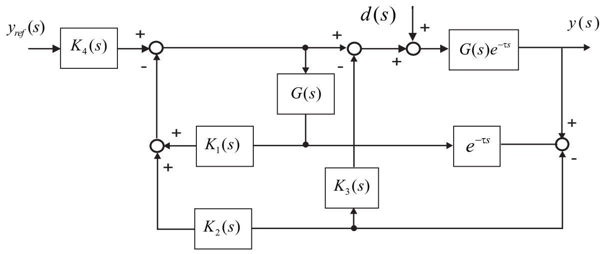

Fig. 3. Xiang DTC structure.

$$y(s) = \frac{K_4 G(s)}{1 + K_1 G(s)} e^{-\tau s} y_{ref}(s) + \frac{G(s) e^{-\tau s} (1 + K_1 G(s) - K_2 G(s) e^{-\tau s})}{(1 + K_1 G(s))(1 + K_3 G(s) e^{-\tau s})} d(s) \quad (9)$$

Remark 5. The closed-loop transfer function from the set-point to the output is given by:

$$G_{ry} = \frac{K_4 \Delta(s) G(s) e^{-\tau s}}{(1 + K_1 \hat{G}(s) - K_2 \hat{G}(s) e^{\hat{\tau}} + K_2 \Delta(s) G(s) e^{-\tau s}}$$

where $\Delta(s) = \frac{(1+K_3 \hat{G}(s) e^{\hat{\tau} s})}{(1+K_3 G(s) e^{\tau s})}$

Remark 6. From the scheme in Figure 3, it can be seen that if $K_1 = K_2 = K_4 = K_r$, and $K_4 = K_0$ this structure is exactly the same as that proposed by (Matausek and Micié (1996)). If $K_1 = K_2 = K_4 = K(s)$ and $K_4 = 0$, the original Smith Predictor approach is obtained.

Consider the unstable model $G_p(s) = k_0 e^{-\tau s}/(Ts-1)$. For a desire closed-loop transfer function for set-point to the output $G_{ry} = e^{-\tau s}/(\lambda s + 1)$, the design parameters are $K_1 = (1 + T/\lambda)/k_0$, and $K_4 = T/k_0\lambda$. For disturbance response, the controller K_3 is designed to stabilize $G(s)e^{-\tau s}$; $K_3 = \sqrt{T/\tau k_0^2}$. In order to achieve a good disturbance rejection, K_2 is a PD controller $K_2(s) = (K_{2d}s + K_{2p})/(K_{2d}s/N + 1)$, with $K_{2p} = T/k_0\lambda$, and K_{2d} dependent on the approximation of $e^{-\tau s}$; $K_{2d} = \frac{\lambda(T - \sqrt{T\tau})(T + \frac{T\tau}{\lambda})}{k_0 \lambda (T - \sqrt{T\tau} k_0 L^2 (\sqrt{\frac{T}{\tau}} - 1)}$, for $e^{-\tau s} \approx (1 - \tau s)$, and $K_{2d} = \frac{\left[\frac{1}{2}L\tau + \lambda(T - \frac{1}{2}L - \frac{1}{2}\sqrt{T\tau}\right](T + \frac{T\tau}{\lambda}) - \frac{1}{4}L^2 \tau (\sqrt{\frac{T}{\tau}} - 1)}{k_0 \left[\frac{1}{2}L\tau + \lambda(T - \frac{1}{2}L - \frac{1}{2}\sqrt{T\tau}\right] + \frac{1}{4}L^2 k_0 (\sqrt{\frac{T}{\tau}} - 1)}$, for $e^{-\tau s} \approx (1 - \frac{1}{2}Ls)/(1 + \frac{1}{2}Ls)$.

Consider the model $G_p(s) = k_0 e^{-\tau s}/s$. For a desired closed-loop transfer function from, $K_1 = 1/k_0\lambda$, and $K_4 = 1/k_0\lambda$, and with respect to stability in the disturbance transfer function $K_3 = \pi/6k_0\tau$. To disturbance rejection, $K_{2p} = 1/k_0\lambda$; $K_{2d} = \frac{(6-\pi)(\lambda+L)}{\lambda k_0 (6-\pi) + k_0 L \pi}$, for $e^{-\tau s} \approx (1 - \tau s)$, and $K_{2d} = \frac{(\frac{1}{2}L + \lambda - \frac{1}{2}\lambda k_0 K_3 L)(1 + \frac{l}{\lambda}) - \frac{1}{4}k_0 K_3 L^2}{k_0(\frac{1}{2}L + \lambda - \frac{1}{2}\lambda k_0 K_3 L) + \frac{1}{4}k_0 K_3 L^2}$, for $e^{-\tau s} \approx (1 - \frac{1}{2}Ls)/(1 + \frac{1}{2}Ls)$.

3. INTERACTIVE TOOL

This section briefly describes the main features of the developed tool programmed with SysQuake (Piguet,

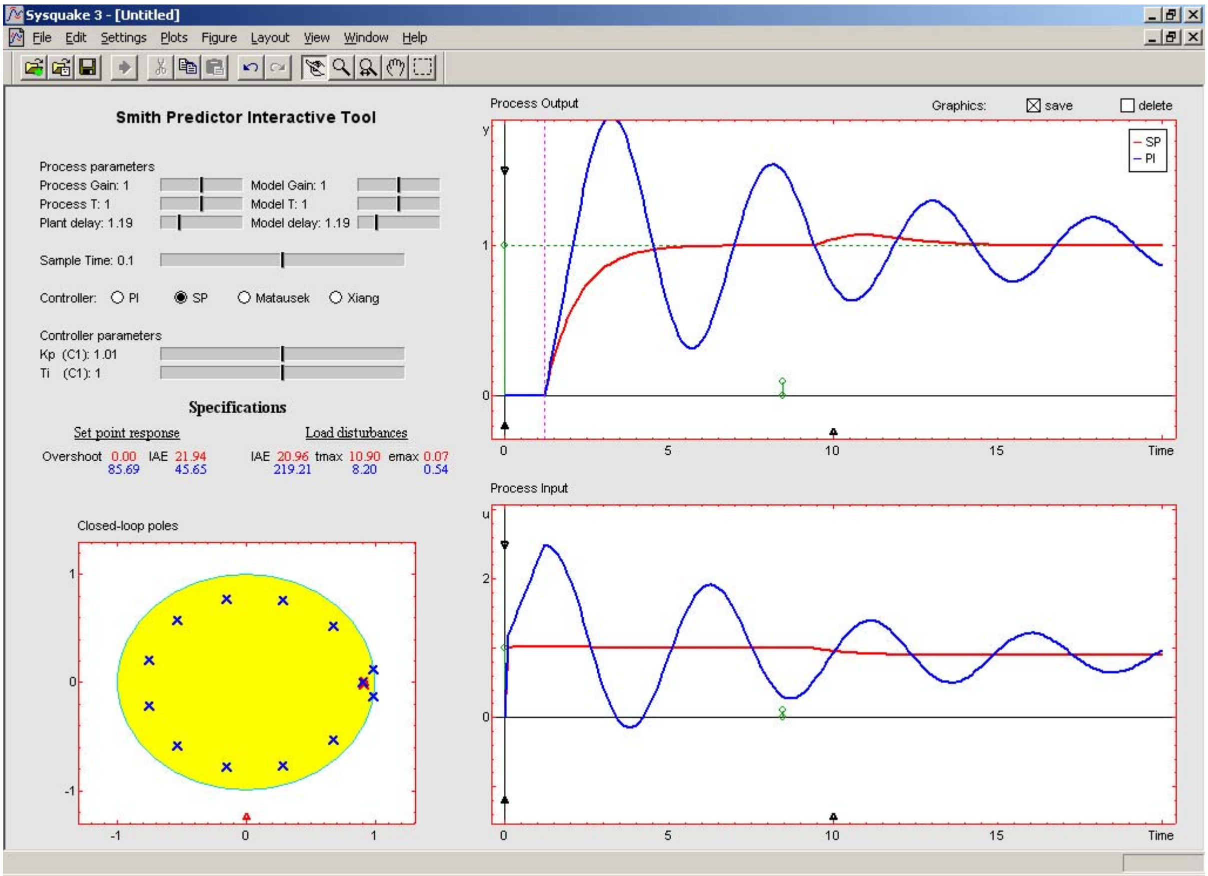

Fig. 4. Interactive tool user interface.

2004). When developing a tool of this kind, one of the most important things that the teacher has to have in mind is the organization of the main windows and menus of the tool to facilitate the student the understanding of the control technique (Dormido, 2004).

The main window of the tool is divided into several sections represented in Figure 4 (basic screen of the developed interactive tool) which are described as following:

- *Graphics*. There exit different graphical elements which represent the system output (Process Output), the controller output (Process Input), and the poles of the closed loop system (Closed-loop poles). The first two graphics show the simulation results of the control algorithm selected for a step change in set-point and load disturbance (in the input). Over these graphics the user can interactively modify the amplitudes of the set-point and the disturbance dragging on the green dots, the time delay of the process dragging on the vertical pink line, and also to change the scales dragging on the small black triangles. The third graphic shows the closed-loop poles of the systems on the unit circle being possible to check and study the stability in an interactive and easy way. On the other hand, the user can use the buttons save and delete at the top right-hand corner of the screen in order to store a simulation for comparison. So, it is possible to save some configuration (for example PI control) and compare it with another one (for example Smith Predictor). The saved configuration is drawn in blue, and the current one in red.
- *Parameters*. The different parameters available in the tools are shown on the left-hand corner of the screen (see Figure 4). There are two kinds of parameters, static ones and interactive ones. The static ones are those shown with the name of Specifications. These parameters characterize performance with respect to set-point and disturbance responses. When two designs are compared using the save and delete options, these parameters are duplicated in order to compare better the performance between them. The interactive ones allow to modify process parameters, the controller type, and controller parameters. So, it is possible to modify the process gain, process time constant, and process time delay. Also, when the Smith Predictor (or one of its variants) is selected, the model parameters can be modified in order to study robustness problems. The different control algorithms discussed in the previous section can be selected allowing to modify the associated parameters in an interactive way. The algorithm has been implemented solving linear differential equations with con-

stant coefficients. So, the sample time is available as an interactive parameter.

- *Settings menu.* Several options can be chosen from this menu. There is one entry called Process Transfer Function where it is possible to choose between the three typical plants used in the study of system with delays: first order system, integrator system, and unstable system. The second entry is called Load/Save options where results can be stored and recalled using the Save design and Load design dialogs. Also, the option Save report can be used to save all essential data in ascii format. The last entry (Autoscale) can be used to choose between and automatic or manual scale for the graphics.

As described above, the interactive tool provides to the user different degrees of freedom in order to study the problem of systems with delay. For example, it is possible to study how a typical PI control has problems to face systems with large delays, and how the Smith Predictor algorithm allows to take into account the delays in an easy way. Also, since the Smith Predictor uses an internal model, the user can study the problems of this algorithm due to the presence of model uncertainties. As well-known, and as commented in section 2, the original Smith Predictor algorithm presents several problems when the plant is described by an integrator or an unstable system. So, the algorithms developed by Matausek and Xiang can be used to study and solve these problems. These are some of the possible exercises that the user can do using the developed tool, where the Load/Save options and save/delete ones provides different ways to store and compare all available algorithms.

In this way, Figure 4 shows an example where a typical PI controller is compared with Smith Predictor. The system is represented by $G_p(s) = \frac{1}{s+1}e^{-1.2s}$ and the controller parameters are $K = 1$ and $Ti = 1$ for both PI and Smith Predictor. It can be seen how the control using an PI becomes more oscillatory obtaining a poorer performance in presence of delay. Next section presents several examples in order to show better the use of the interactive tool.

4. EXAMPLES

Example 1. Consider the integrator process:

$$G_p(s) = \frac{1}{s}e^{-2s}$$

For instance, if the control goal is supposed to place the process closed-loop poles around $s = -0.6$, the controller from Smith Predictor scheme is given by $K(s) = 0.6$.

For the same desired closed-loop poles the controller from Matausek scheme will be $K_r = 0.6$. To reject load disturbances, from equation (8) $K_0 = 0.1$.

The responses of the two schemes are shown in Figure 5 (blue for Matausek scheme, and red for Smith Predictor one). Note that the Smith Predictor approach has steady-state error for load disturbances (see equation (5)) while for the Matausek one the error is zero.

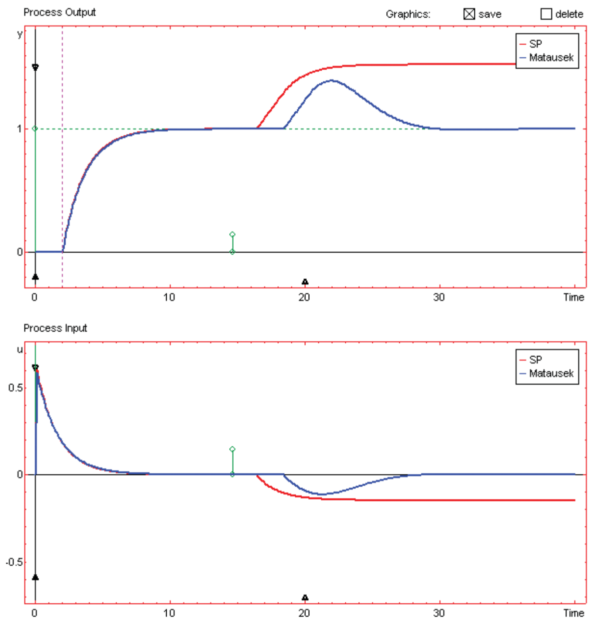

Fig. 5. Output response of the Smith and Matausek schemes for a step load disturbance of 0.1 at $t = 15$.

Example 2. Now, let us consider the unstable process:

$$G_p(s) = \frac{4}{(10s-1)}e^{-2s}$$

If the control goal is to place the process closed-loop poles around $s = -0.1$, the controller for Smith scheme is $K(s) = 0.5$. The response of the closed-loop system for ideal modelling is shown in Figure 6 in blue. Note that the systems becomes unstable even without modelling error (see Remark 1).

For the Xiang scheme, if the same desired closed-loop features are considered, $K_1 = 1.5$, $K_2 = \frac{7.93s+1.25}{\frac{7.93s}{10}+1}$, $K_3 = 0.35$ and $K_4 = 1.6$ are obtained. Figure 6 shows (in red) that the response of the closed-loop system is stable.

Example 3. Consider the same example described in Figure 4, where the real plant is given by

$$G_p(s) = \frac{1}{(s+1)}e^{-1.2s}$$

In order to analyze the robustness of the PI controllers and Smith Predictor schemes, let us considered that there is a modelling error such that the model is described as

$$\hat{G}_p(s) = \frac{1}{(s+1)}e^{-2.6s}$$

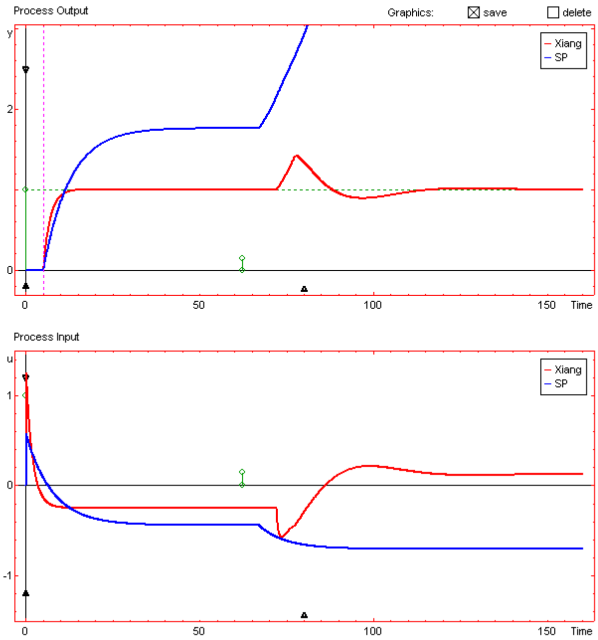

Fig. 6. Output response of the Smith Predictor and Xiang schemes for a unstable system and load disturbance of 0.1 at $t = 55$.

The results are shown in Figure 7 where it can be seen how the Smith Predictor scheme is unstable while the PI controller remains stable (although with a poor performance).

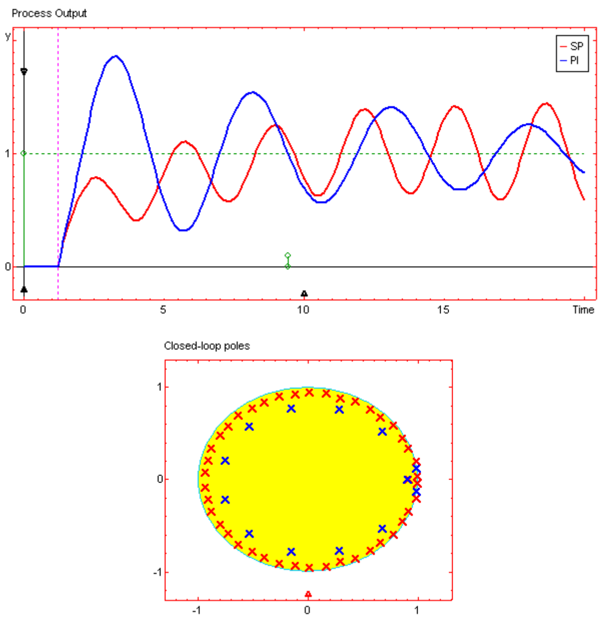

Fig. 7. Output response and closed-loop poles.

5. CONCLUSIONS

An interactive tool which allows to study the problem of systems with large delays has been presented. The developed tool provides several examples in order to compare performances of PI controllers and Smith Predictor approaches for stable, integrating and unstable systems with delay. Robustness problems due to presence of model uncertainties can also be studied.

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